Excerpt from

Technology Meets Math Education: Envisioning A Practical Future

Forum on the Future of Technology in Education

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TECHNOLOGY MEETS MATH EDUCATION: ENVISIONING A PRACTICAL FUTURE FORUM ON THE FUTURE OF TECHNOLOGY IN EDUCATION

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It is the presupposition of this workshop that computers are here to stay—both inside schools and out—and that significant changes in the way we teach and learn will result from their presence. Far be it for me to question that assumption. But in the midst of the exhilaration of being visionary, I want to temper our expectations with a practical view of where schools and classrooms are now and how much they might change in the next decade. I'm certainly not a Luddite and my intent is not to throw cold water on our flights of fancy, but I firmly believe that simultaneously considering both the vast potential of technology and the current realities of schools can lead us to creative solutions to problems we might not otherwise have considered. In this situation, practicality and therefore necessity, may indeed be the mother of invention.

The pairing of mathematics and technology has a long history; for many years a knowledge of mathematics was considered a prerequisite for becoming a programmer and the use of computers was thus available primarily to a small group of mathematically-inclined enthusiasts. Even now, when many people use computers for writing and communication, math-related programs occupy one third of a recent catalogue of educational software (Sunburst, 1999)—much more than any other topic. But this association may have led us down the wrong path, where we've seen computers primarily as machines for calculating and, educationally, for presenting students with exercises in calculation.

This paper insists that, rather than looking at math education from the perspective of the computer, we must look at computers from the perspective of mathematics education. The primary tenet of this paper is that the role of technology in math education must be in service of goals we hold for student's mathematical knowledge and expertise. Of course, technology may dramatically change those goals as well (which could get us in a serious infinite loop), but it is still the aims of mathematics education to which we must return. Broadly speaking, I take these goals to be (based on the NCTM Standards):

- eveloping students' "mathematical literacy" that goes far beyond arithmetic computation—e.g. a thorough knowledge of our number system that underlies computation and estimation, a facility with data that supports the critical analysis of statistical information with which we are bombarded, a comfort with geometric analyses of space, both two and three dimensional, an understanding of what different representations of mathematical quantities—such as graphs—mean and how they relate;

- Supporting students' mathematical "habits of mind," e.g. ability to engage in mathematical proof and argument as a basis for logical thought and discussion;
- Preparing students for the judicious and effective use of computational tools and technologies;
- Nurturing a positive attitude toward and curiosity about mathematics and mathematical thinking that can serve as the basis of lifelong learning;
- Empowering students with the realization that mathematical knowledge (not to mention much else that we learn in school) does not come predigested from teachers and books, but is a product of their own thought and exploration.

These goals take us far beyond the arithmetic that often occupies the majority of elementary school mathematics education and beyond, as well, the formulaic approaches to algebra ("solve this polynomial") and geometry ("prove this theorem") with which many of us are familiar. These goals are best served by *the creation of communities of learners in which students are actively engaged in the process of mathematical sense-making.* In this paper, the promise of technologies will be measured against this vision.

In this context, we can see that the present and future roles of technology in math education are both powerful and problematic; we need to paint a picture that takes advantage of the potential of technology without falling into the technology = computation trap. There are indeed many significant opportunities that go far beyond this impoverished image and I will describe several below. But it is important to note before jumping into descriptions of several compelling uses of technology in math education (says the realist), that the existence of these opportunities does not guarantee that they will be used effectively—or at all. The effects of technology on education and on society in general are emphatically sociotechnical (Bruce, 1999), that is, the technology has an effect only through people's uses and attitudes, in this case, in particular, through pedagogical philosophy. Technology in a vacuum is just that—technology in a vacuum. We will need to figure out how to create the context that will allow this potential to be realized.

The seeds of most of the potential future uses of technology in math education are present in today's possibilities, although we are just beginning to learn how to take advantage of them. In the following sections, I will discuss several categories of technology use, noting the present situation and future possibilities. The structure of the rest of this paper will be:

1. Descriptions of five powerful uses of technology in math education, present and future;
2. A consideration of the factors that are necessary to fulfill this potential;
3. Some concerns about the integration of technology into math education;
4. A brief closing restatement of the dilemma

POWERFUL USES OF TECHNOLOGY IN MATH EDUCATION, PRESENT AND FUTURE

As a way of organizing the ways in which technology may have substantial and significant effects on mathematics education, I have chosen five types of opportunities afforded by computers, calculators, the Internet/Web, and associated input and output devices. In each case, I will give examples of present uses, note how they support the goals identified and project how uses of this technology might grow in the near future.

DYNAMIC CONNECTIONS

Mathematics is most often thought of as an "abstract" topic, populated by symbols and invisible concepts. For many students, this lack of a visual representation makes it difficult to make connections between a mathematical expression and the situation to which it refers. Technology can help here; computers, in spite of their early image as calculating machines, are decidedly visual and provide a medium in which visual representations can be made dynamic. Students do not have to be stuck with a description in words and symbols OR with a diagram in a book that that can't be examined or explored. Here are two examples of the difference technology can make.

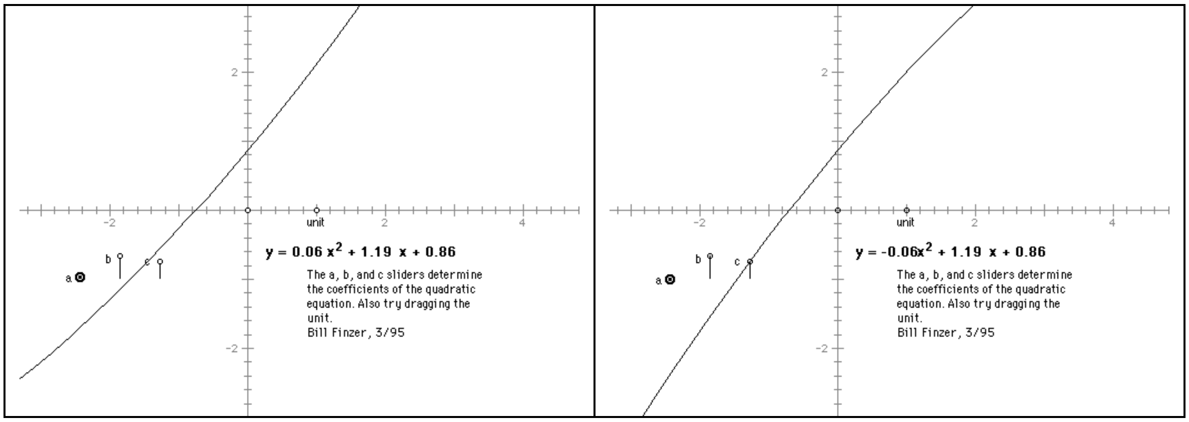
Most students' notions of geometry are, at worst, of two-column proofs that follow a series of arcane rules, illustrated by one or two static line drawings. Many students who enjoy and succeed in geometry are able to supplement these pictures with some sense of motion, e.g.: if this corner of the square moves here, that angle will grow twice as big. The computer allows everyone to visualize these changes. Several pieces of "dynamic geometry" software have dramatically changed possibilities for geometric exploration. These tools go a long way toward turning mathematics into an experimental science—much closer to the way mathematicians experience mathematics than students usually do.

Figures 1a and b show two views of a dynamic diagram from the Geometer's Sketchpad that allows a student to explore quadratic equations (containing the square of one variable) and the curves that they define. The power of this diagram lies in the fact that the student can change the curve by moving any of the three sliders on the lower left to change the value of a, b, and c. As the sliders are moved, both the equation and the graph change at the same time, so the relationship between them is visually apparent. It is precisely this dynamic linking that makes this software powerful. There are many explorations possible using this particular diagram; here is one simple one. Figure 1a shows a parabola whose first coefficient is .06. Note that the curve is almost straight. Figure 1b is a similar parabola whose first coefficient is -.06. This curve is also almost straight, but curves in the opposite direction. The student can move the slider back and forth between these two (and beyond), watching how the curvature changes—and what happens when the coefficient becomes 0—the curve becomes a straight line! To most students, parabolas and straight lines aren't related; after all, one curves and the other doesn't—but this dynamic diagram shows that a straight line is just a parabola with a 0 coefficient.

Not only can computers draw graphs and other mathematical objects and allow students to "play" with them, they can relate them to images in the "real" world. One way these connections can be made is with digital cameras and videocameras; no longer are the pictures we take static objects, but as digital objects they take on a new life that enables them to be closely linked with mathematical representations.

FIGURE 1a and b

Dynamic Geometry Diagrams To Investigate The Relationship Between Parabolas and Straight Lines (Among Other Things)

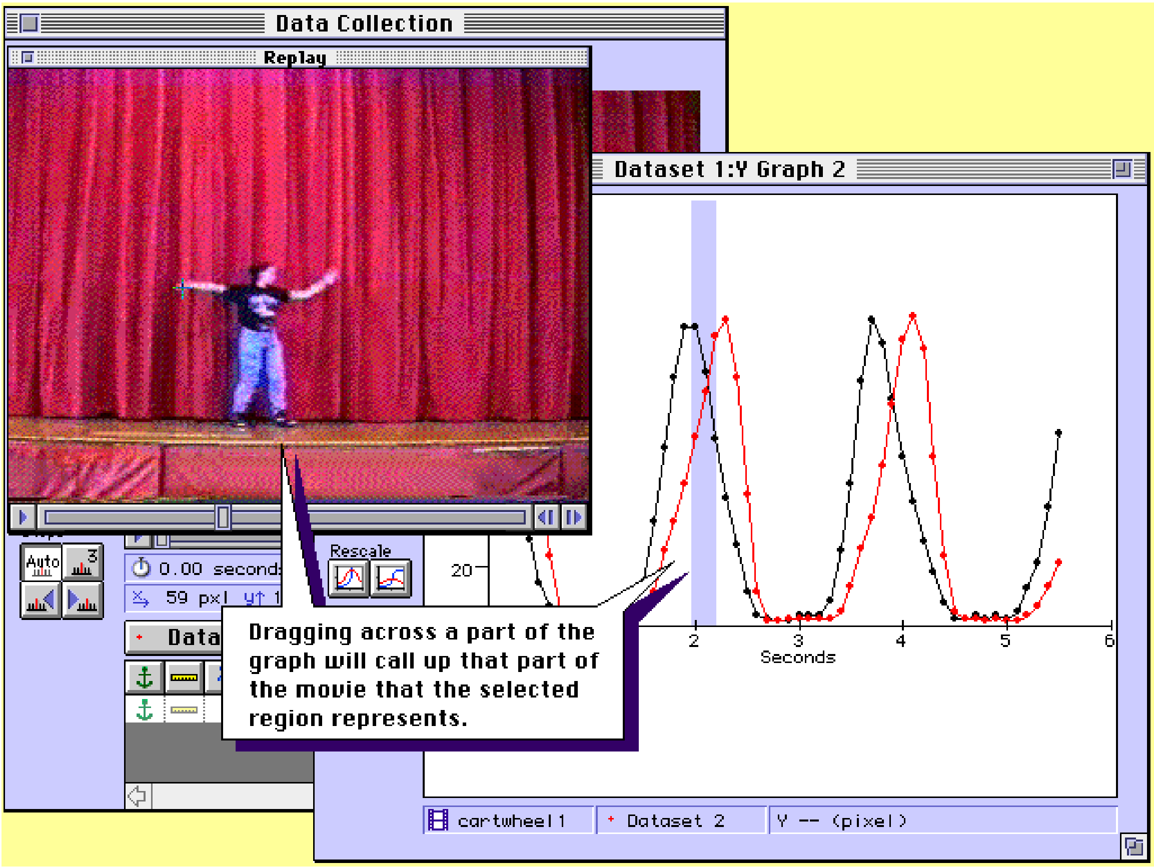


One such piece of software is CamMotion (Rubin and Boyd, 1997). CamMotion (a play on Camera and Motion) provides tools to analyze motion as it is captured on a video camera and to create the corresponding graphs of changes in position or speed over time. First, the video is digitized, so that it can be displayed on and controlled by the computer. Then, the student chooses an object to track (e.g. someone's hand) and clicks on it in each frame as the computer plays the video one frame at a time. The data thus gathered can be seen as a graph; speed can be calculated from the distance between adjacent data points and concepts such as acceleration and deceleration can be explored. But perhaps the most important aspect of this software is that the video and the graph are linked, that is, when the student points with the mouse to a point on the graph, the corresponding frame of the video is displayed. Similarly, when the student plays the video, the corresponding points on the graph are highlighted. So the link between the "real world" and its mathematical representation is made visible in a way that is quite striking.

Figure 2 shows an example of CamMotion being used to analyze the motion of a girl doing a cartwheel. The student has clicked on the position of the girl's left and right hands for each frame of the video, than made this graph by displaying the height of her left hand and the height of her right hand on the same graph. You can see that each hand follows a similar path going up and down, with one hand ahead of the other. The displayed video frame corresponds to the highlighted part of the graph; both of her hands are at about the same height, midway between the lowest (floor) and highest points they reach. It is also possible to see how quickly her hands are moving from this graph. When is her hand moving most quickly? Most slowly? How could you tell from this graph?

FIGURE 2

A Cammotion Graph of The Motion of a Girl's Hands Doing a Cartwheel



Embracing a student's real world like this is especially important when we consider technology's place in math education, because the general effect of computers is to separate students' concrete experiences from their digital representations. Being able to capture students' physical experience is more than just a new "input device;" it allows us to turn some of our mathematics pedagogy on its head. As just one example, imagine if the student's task is to move in a way that matches a particular graph, which, in turn, may have come from another video—we might call this life imitating math imitating life.

As digital devices make it increasingly simple to capture representations of the analog world on the computer, there will be more opportunities to treat the world as a grand data base, whose secrets and rules are waiting to be discovered. These systems provide a more "intimate" connection to the mathematics that can counteract the general effect of computers to separate students' concrete experiences from their digital representation, In addition, they create an environment in which mathematics is an experimental science, in which trying things out and noting what happens is an acceptable—and even preferable—approach. Having shared mathematical representations—displayed on a screen that is visible by several students at a time—also supports collaboration, since it provides objects to refer to, talk about, and investigate and goes a long time toward creating a mathematical community.

SOPHISTICATED TOOLS

Many authors have noted the growing importance of numeracy in our lives; few jobs are immune from a need for mathematical sense-making. As a result, many workplaces now provide workers with an integrated tool set (spreadsheet, calculator, graphing calculator, graphing/data analysis tools) and expect that they will have the expertise to use them effectively. Part of the responsibility of math education is to "keep up" with these developments in order to prepare students for the work world. Having such a set of tools widely available to students also has the potential to significantly change the curriculum—to give students access to mathematical topics and insights by removing computational barriers to inquiry.

This is an area of some controversy; many people who grew up mastering pencil-and-paper algorithms fear that if students use calculators they will never learn the basics of computation and will be lost without this tool. In fact, in many ways, the opposite is true: knowing how to use a calculator appropriately requires the student to know which numbers to enter, what operations to carry out and how to interpret the answer—all more important and often more demanding than doing the calculation itself. There is plenty of evidence that the <u>appropriate</u> use of calculators can improve students' mathematical achievement, as well as lead to more positive attitudes toward mathematics. In addition, calculators can add significant richness to students' mathematical experiences. Here are some examples:

The Range Game, developed by Grayson Wheatley, asks students to start with a number (say 37) and, using a calculator, find numbers that when multiplied by it give a number in the range from 500 to 600. In the conversation reported by him, students talk about their estimates, their results, whether they "have them all," and the largest and smallest numbers that would work (which leads to a discussion of decimals and limits.) This kind of conversation, which exercises students' number sense and even leads them into unfamiliar mathematical territory (e.g. limits), would be impossible if calculators were not available.

Graphing calculators have been more consistently praised as enhancing mathematics education. The ease with which they can produce complete pictures for a variety of functions means that students can graph functions, zoom in for greater detail, zoom out to see the function as X increases or decreases and compare graphs of one function with those of others. Simple models—e.g. of population growth—can be built and run on these calculators. In essence, much of the power of programs that a few years ago ran on microcomputers has been captured on personal, portable, affordable technology. Palm Pilots are the latest example of these personal aids; one of the most exciting uses of these hand-held computers is as a data collection device that can go where the student goes, rather than being stuck in a classroom.

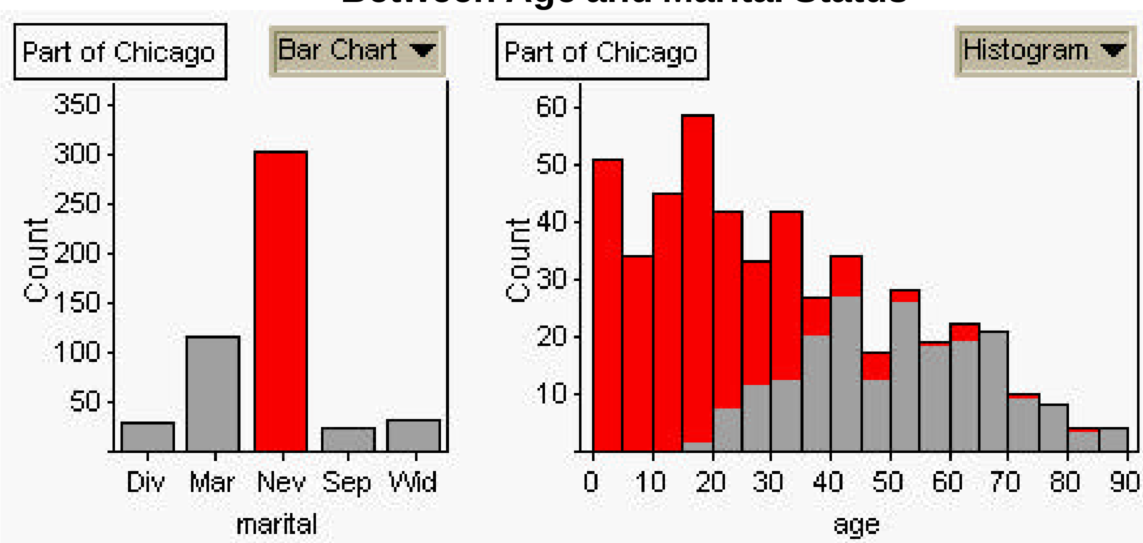
The use of calculators makes possible significant changes in the mathematics curriculum. While it is still important for students to understand computation and be fluent in carrying out problems of reasonable size, there is no reason for students to spend time dividing 5-digit numbers, adding long columns of numbers or finding square roots. The emphasis can instead be on problem-solving and developing number sense. Graphing calculators call into question the emphasis on algebraic symbol manipulation; many of the skills students learn symbolically (e.g. factoring complex polynomials to figure out the roots) are more effectively taught by working with the graphs of the functions.

In addition to calculators, other tools have the potential to change the math curriculum. Sophisticated data analysis tools, for example, expand what students can learn about statistical reasoning. The newest tools do more than produce fancy graphs; they facilitate the discovery of patterns through exploratory data analysis. These tools are well-suited for complex data bases with many variables and employ new kinds of graphs, many of which are interactive in ways similar to the dynamic geometry software described above. Many of these tools are actually used by statisticians, and several are designed with educational purposes as well. One such tool is Fathom, a sophisticated tool that provides students with many ways to look at—and therefore understand—complex data bases. Here is a relatively simple example, illustrated in Figure 3:

The data set is of a large number of people in Chicago—two of the variables are age and marital status. To see how old people are in each marital category (divorced, married, never married, separated, widowed), we can select that bar in the left hand bar graph and all the matching people are highlighted in the histogram graph on the right; this graph shows that most (but not all) of the people who have never been married are young. We can investigate related questions by choosing other bars in the bar graph (e.g. how old are the people who are divorced?) or by choosing one or more of the bars in the histogram and seeing how people in that age group are distributed among the marital categories. (e.g. What portion of people in the 50–60 age category are divorced?) In general, Fathom makes it easy to do intelligent data analysis: the student can research a question with a few simple commands and the resulting graphs will provide at least a partial answer to the question—and inevitably pose additional ones.

FIGURE 3

Fathom Graphs Exploring the Relationship Between Age and Marital Status



The value of such tools is that they create a boardwalk over the computational swamp, allowing students to see patterns they would never glimpse if they had to do the calculations or even draw the graphs themselves. In this way, exploratory data analysis software (and other visualization techniques) play a similar role to calculators and graphing calculators in emphasizing the meaning in mathematical objects and the beauty of the patterns they exhibit.

RESOURCE-RICH MATHEMATICAL COMMUNITIES

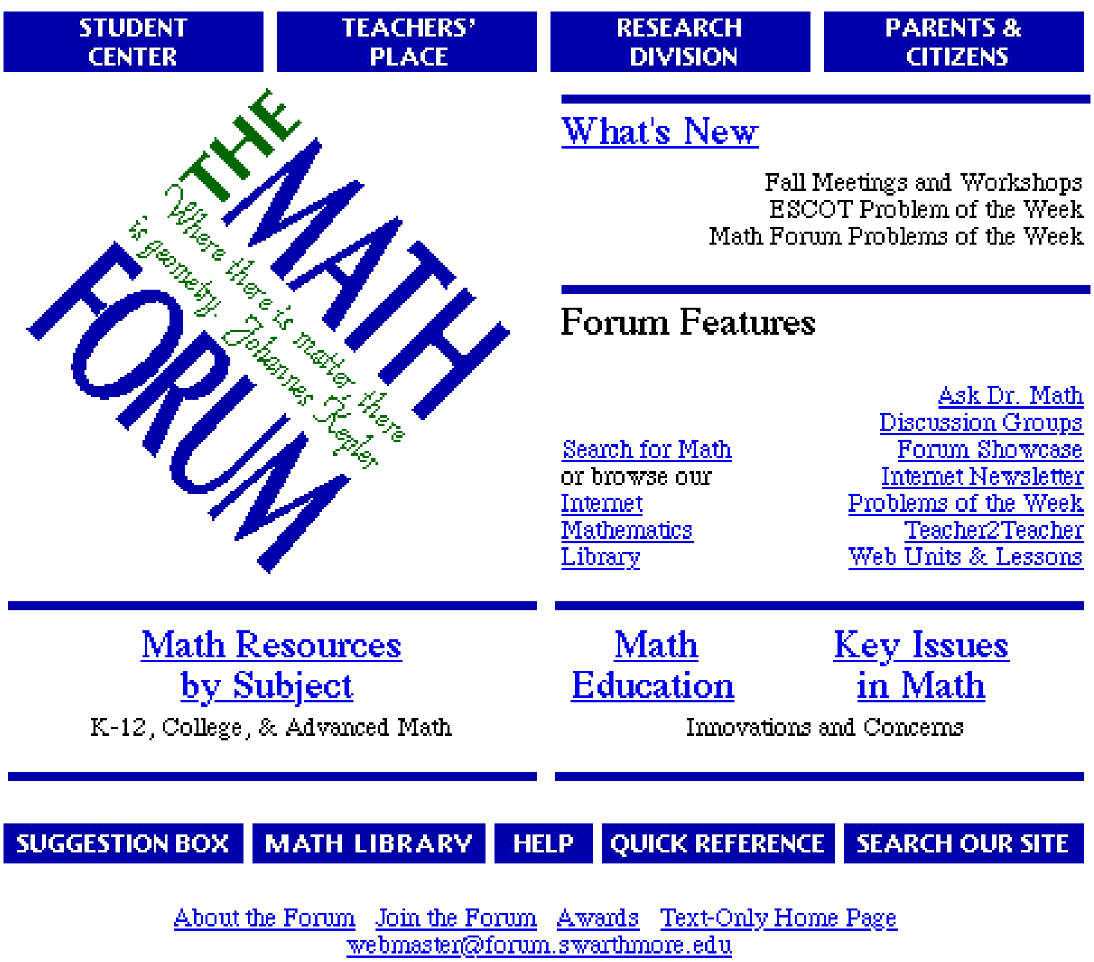
More than any other recent development, the Web has changed the public's view of technology; while few people had even heard of the Web five years ago, it is now almost impossible to watch a television ad that does not mention a Web site. The amount of information available on the Web continues to expand exponentially as more and more diverse organizations—profit-making, non-profit, large, composed of one person—are getting into the act.

This extraordinary growth has led to several developments that have important implications for mathematics education:

Resource sites. The best known of these is the Math Forum (www.mathforum.org), whose home page is shown as Figure 4.

FIGURE 4

The Home Page of the Math Forum



The Math Forum site includes a large list of (screened for quality) resources for K through college math teaching, including interactive activities; recommendations of software; examples of classroom activities and links to related discussion groups; a conversation space for teachers (Teacher2Teacher); an extensive Internet Math Library, which contains even more resources than the resource list; an Ask Dr. Math feature, in which an expert answers students' questions (the most recent topic is rounding); Problems of the Week at a variety of levels of difficulty; discussion groups on topics such as discrete math and a multi-lingual discussion on the history of mathematics; A Forum Showcase that highlights recently added sites, e.g. the following:

> Exploring Data—Pages for finding and displaying data sets, designed to support workshops on statistics given by the Math Forum for the Urban Systemic Initiative (Philadelphia and San Diego). Included are: links by level to relevant statistics Standards (NCTM, California, Philadelphia); lesson plans for collecting, analyzing, and/or displaying data; sources of data sets, general information, courses, and statistics software on the Web; and an "Oceans of Data" page with a data set (diving records) to download, instructions for making ClarisWorks graphs, suggested questions for discussion, and related 'ocean links' (NOAA, SeaWifs, tide tables, etc.). and on-line exhibits, e.g. of symmetrical patterns in Oriental carpets.

The site is impressively complete and well-organized and continues to grow as more materials are produced. It has served as an important portal for mathematics educators and as a kind of social center for the mathematics education community.

On-line professional development. In addition to the professional development materials included on sites like the Math Forum, there are entire courses being developed for delivery online. Lesley College in Cambridge, for example, teaches a semester-long online course on technology in education (much of which is focused on mathematics) for pre- and in-service teachers. Other organizations have put together repositories of professional development materials, some of which contain digitized video segments of classrooms and interviews with teachers. This material is of varied quality, of course; being on the Web is no guarantee that something has been well-designed, but it is much easier for teachers to find the resources and judge for themselves than it would be if the materials had to be ordered.

Mathematical communities for students. One of the hopes for the Internet and Web is that they would provide students with a sense of community and audience for the math they are doing—and that these communities might even be international. There have been some successes in this regard (e.g. the Problem of the Week on the Math Forum). While these uses have the potential to engage large numbers of budding mathematicians, they require some human infrastructure to organize students' participation and their impact is still unclear.

Possibility of home-school connections. Another vision put forward for the Web is the possibility that it could enable homes and schools to communicate more effectively. Possible scenarios that have been proposed are: parents and teachers could now have access to the same information about students and might communicate via email; schools might post homework on the Web or send comments to parents about children's work; schools might post materials for parents to use with their children at home to reinforce what they have done in school. There is potential here, but the uses suggested so far don't seem to make a significant leap from the status quo and few of them have been implemented in enough places to assess their effect.

Availability of data. The Web opens up a huge world of data to every student. Data bases that one could never imagine accessing before (seismographic data, weather data, environmental data) are now out there for the taking. Some Web sites have even been created with the express purpose of compiling and supplying databases for statistical analysis (e.g. the CHANCE data base out of Dartmouth). Students can also participate in creating large databases with information culled from a variety of classrooms; if these data are correlated with some geographical variable, students have a particular interest in comparing their data with that from distant schools. The National Geographic Kids Network was one of the first such projects; measurements of the pH of rainfall and local bodies of water were analyzed to track the sources and effects of acidity. Other examples have been: classroom air quality, butterfly migration studies, and measurements of shadows from different places at the same time. These kinds of projects make students members of a larger mathematical community and give them the opportunity to engage in a mathematical activity that reflects what real scientists do.

The Web will only get larger, with faster connections and more information. We can anticipate that as the amount of material on the Web increases, the difficulty of sifting through all the resources will increase as well. In addition, many of the powerful uses of the Web require human infrastructure as a foundation—organizing a coordinated data collection activity or a math competition must begin with personal contact (albeit over email) that can then make the best use of the Web's capabilities. Getting schools connected to the Internet has been a major policy goal for the past several years; now that we've come a long way toward achieving that goal, it's time to look more critically at the possible uses we might make of these electronic connections.